Quantitative Analysis of Climate Heterogeneity via an Unconditional Quantile Vector Error Correction Model

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(Ph.D. Student in the Job Market)

Introduction

- Climate change dynamics are non-uniform across space and time. Different forms of heterogeneity can be captured through the **unconditional quantiles of temperature** converted into **time series objects**.

- Accounting for climate heterogeneity in economics studies is crucial for correctly anticipating the non-linear consequences of climate change and for informing optimal mitigation and adaptation policies.

- This paper introduces a time-series methodology to study different forms of heterogeneity in the dynamics of the unconditional temperature distribution and its association with climate drivers.

- **Vector Error Correction Model (VECM)** for the **unconditional distributional characteristics of temperature** (mean and quantiles) and **radiative forcing**, including radiative forcing from greenhouse gases (CO2, methane, etc.). Motivated by the physics theory from **Energy Balance Models (EBMs)**.

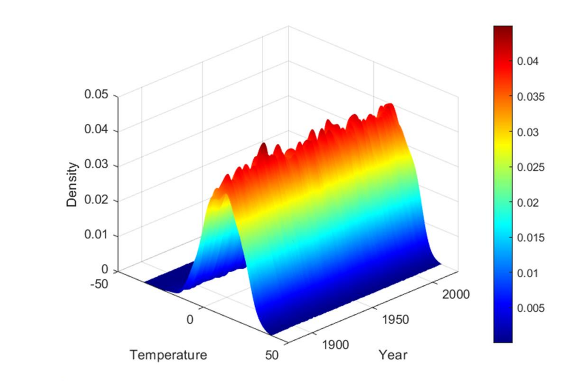

Figure 1. Unconditional densities of temperature 1880 - 2021, North Hemisphere (NH)

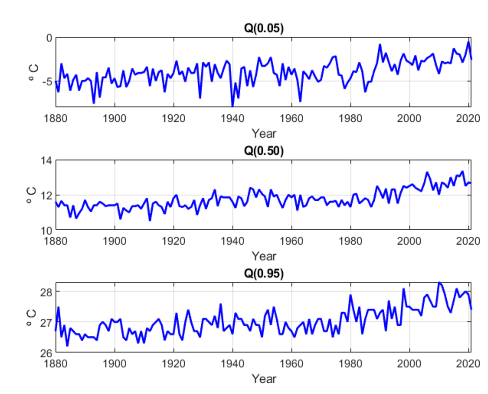

Figure 2. Unconditional distributional characteristics, NH

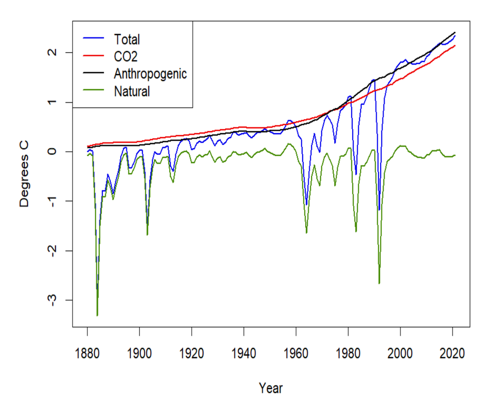

Figure 3. Radiative Forcing

- Research outcomes of practical interest for economic analyses:
  - Estimation of **distributional climate sensitivities.**
  - **Common-trending component** extraction.
  - **Long-term density forecasts** of temperature.
  - Identification of **distributional structural shocks** and impulse-response analysis (different from standard average shocks).
  - **Conditional projections** of temperature distribution under hypothetical **greenhouse gases emissions scenarios.**

- Alternative outcomes to those produced in climate science using **General Circulation Models (GCMs)**, but obtained in a simpler, less time and computational consuming, reduced form approach.

One-Dimensional EBM and the VECM

- Energy Balance Models (EBMs) are climate models used to describe the **change in temperatures as a function of incoming and outgoing radiation**.

- Following Held and Suarez (1974), at a given latitude θ_i, a simple EBM is expressed as:

$$C(\theta_i)\frac{\partial T_t(\theta_i)}{\partial t} = [F_t - \lambda(\theta_i)T_t(\theta_i)] + \gamma(\theta_{ii})[\overline{T}_t - T_t(\theta_i)] + \sum_{j\neq i}\gamma(\theta_{ij})[\overline{T}_t - T_t(\theta_j)], \quad (1)$$

where $\overline{T}_t$ is the average temperature, $T_t(\theta_i)$ is latitudinal temperature, and F_t is the total radiative forcing, including radiative forcing from greenhouse gases.

- Noticing that **at global or hemispheric scales, temperature unconditional quantiles represent temperatures at different latitudes**, and relying in a first order discrete time approximation for stochastic processes, (1) can be written as:

$$\Delta Q_t(\tau_i) = \frac{1}{C(\tau_i)}[F_{t-1} - \lambda(\tau_i)Q_{t-1}(\tau_i)] + \frac{\gamma(\tau_{ii})}{C(\tau_i)}[\overline{T}_{t-1} - Q_{t-1}(\tau_i)] + \sum_{j\neq i}\frac{\gamma(\tau_{ij})}{C(\tau_i)}[\overline{T}_{t-1} - Q_{t-1}(\tau_j)] + u_t(\tau_i), \quad (2)$$

where $Q_t(\tau_i)$ is the quantile associated to $T_t(\theta_i)$. **Each equation has the form of a restricted VECM**.

Climate Econometrics Methodology

- Time series methodology to estimate the unconditional-quantile VECM:

  1. Estimation of annual unconditional distributional characteristics (mean and quantiles) of temperature **using historical data**.
  2. Testing for the **co-trending rank** using the approach by Guo and Shintani (2013).
  3. Estimation and testing of **co-trending vectors** using the approach Chen et al. (2022), **being agnostic about the type of trends in the data** (stochastic or deterministic).
  4. Estimation (equation by equation) of the short-run dynamics in the VECM by OLS:

$$\Delta Q_t(\tau_i) = \frac{1}{C(\tau_i)}[F_{t-1} - \overline{\lambda T}_{t-1}(\tau_i)] - \frac{\gamma(\tau_{ii})}{C(\tau_i)}[Q_{t-1}(\tau_i) - \hat{\beta}(\tau_i)\overline{T}_{t-1})] - \sum_{j\neq i}\frac{\gamma(\tau_{ij})}{C(\tau_i)}[Q_{t-1}(\tau_j) - \hat{\beta}(\tau_j)\overline{T}_{t-1}] + v_t(\tau_i). \quad (3)$$

- **Contribution:** Agnostic co-trending model produces an error-correction mechanism (à la Engle-Granger):

$$y_t = \beta x_t + u_t, \quad u_t = \rho u_{t-1} + e_t, \quad (4)$$

$$x_t = g_t + \epsilon_t, \quad \epsilon_t = \alpha u_t + v_t, \quad (5)$$

where g_t is a trending component. **The model can be written in error-correction form as**:

$$\begin{pmatrix}\Delta y_t \\ \Delta x_t\end{pmatrix} = \begin{pmatrix}\beta \\ 1\end{pmatrix}(g_t - g_{t-1}) + \begin{pmatrix}(\rho-1)(1+\beta\alpha) \\ \alpha(\rho-1)\end{pmatrix}(y_{t-1} - \beta x_{t-1}) + \begin{pmatrix}(\beta\alpha+1)e_t + \beta\Delta v_t \\ \alpha e_t + \Delta v_t\end{pmatrix}. \quad (6)$$

Empirical Analysis

- **Geographical Scale:** North Hemisphere
- **Vector of variables:** $Z_t = [Q_t(0.05), Q_t(0.50), Q_t(0.95), \overline{T}_t, F_t]'$
- **Data:** Station-level temperature data from Climatic Research Unit (CRU) and radiative forcing data from Hansen et al (2011)
- **Sample period:** 1880-2022
- **Main findings:**
  - Co-trending rank equal 3, **one common-trending component.**
  - Heterogeneous climate sensitivities across the temperature distribution (Figure 4): **greenhouse gases affect more the lower quantiles.**
  - Matrix of adjustment coefficients (Table 1) consistent with climate theory.
  - Orthogonal complement of adjustment coefficients matrix used to **obtain the common-trending component**, CT_t (Figure 5), behind the existing warming trend:

$$CT_t = -0.11 * Q_t(0.05) - 0.02 * Q_t(0.50) + \mathbf{0.26} * Q_t(\mathbf{0.95}) + \mathbf{0.88} * \overline{T}_t + \mathbf{0.50} * F_t. \quad (7)$$

  - Projections of the temperature distribution under hypothetical Shared Socioeconomic Pathways (SSP) scenarios of future emissions (Figure 6). **Similar to IPCC projections using GCMs.**

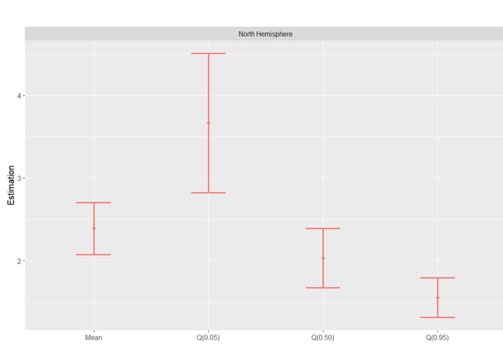

Figure 4. Distributional climate sensitivities, NH

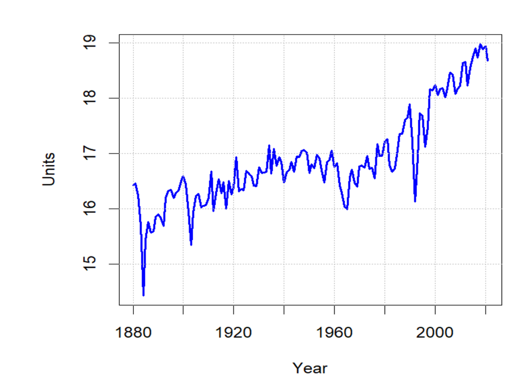

Figure 5. Common-Trending Component, NH

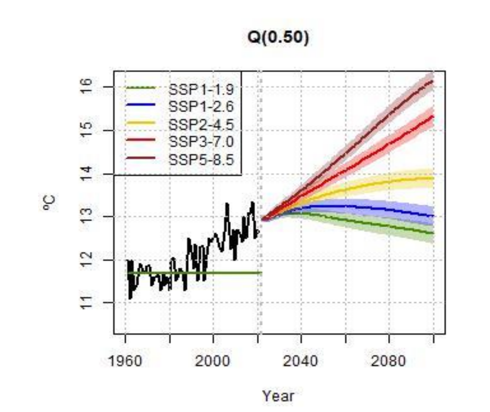

Figure 6. Projections under SSP scenarios

Variables	$\Delta Q_t(0.05)$	$\Delta Q_t(0.50)$	$\Delta Q_t(0.95)$	$\Delta\overline{T}_t$	ΔF_t
$e_{1,t-1} = Q_{t-1}(0.05) - \beta(0.05)\overline{T}_{t-1}$	-0.7675***	-0.0306	-0.0766	-0.02968	-0.0845
$e_{2,t-1} = Q_{t-1}(0.50) - \beta(0.50)\overline{T}_{t-1}$	0.5742	-1.0451***	0.0691	0.0510	-0.0327
$e_{3,t-1} = Q_{t-1}(0.95) - \beta(0.95)\overline{T}_{t-1}$	0.5277	0.1986	-0.8050***	0.2444	0.1421
$e_{4,t-1} = F_{t-1} - \overline{\lambda T}_{t-1}(\tau_i)$	-0.6350**	-0.3230***	-0.2647**	-0.3583***	0.5715***

Table 1. Adjustment coefficients matrix of the VECM

- Results robust to inclusion of more quantile series and climate variables (sea temperatures and ice coverage), sample window (1959-2022), and measures of radiative forcing (CO2 and anthropogenic). Results available at other geographical scales: Global, Europe, Central England.

Discussion and Further Research

- Observational time-series studies and GCMs used in climate science are complementary:
  - Independent observation-based evidence.
  - Comparable estimation and projection outcomes obtained in a **simpler less time-consuming reduce-form approach.**
  - Provide **an alternative measure of uncertainty** based on the residual variance.
  - Often have a better forecasting performance than theory-based models.

- Research outcomes from this paper can be integrated into economic studies of climate change:
  - Calibration and uncertainty analysis of the climate module in integrated assessment modeling.
  - Projection and forecasting of economic damages at global and local scales accounting for distributional heterogeneity.

- **Several avenues for future research:**
  - More granular regional analyses of climate heterogeneity at various levels, such as continental, latitude-belts, country, or sub-regional. Helpful to inform local adaptation and mitigation policies.
  - Inclusion of socioeconomic factors such as GDP, Poverty, and Inequality to explore the interplay between climate dynamics and economic variables.
  - Identification of shocks across the temperature distribution to extend the recent literature on the macroeconomic impacts of climate change (Bilal and Kanzig, 2024; Nath et al., 2024).

Conclusions

- **Climate heterogeneity is crucial** for attribution analysis and the characterization of local damages and optimal climate policies.
- Need for **(beyond the mean)** econometric methodologies to quantify different forms of climate heterogeneity to complement and enhance existing economic analyses.
- The proposed Unconditional-Quantile VECM is an useful reduced-form multivariate statistical alternative to complex large-scale GCMs. Able to **produce responses to key estimation and forecasting queries from climate science within economics**.
- Robust methodology to the type of trends and flexible to **capture both geographical and seasonal heterogeneity**.
- Opens the door to a **new class of economic research focused on the climate distribution** to better inform policy-decisions.

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